Comment on "1/f noise in the Bak-Sneppen model"

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Contrary to the recently published results by Daerden and Vanderzande [Phys. Rev. E **53**, 4723 (1996)], we show that the time correlation function in the random-neighbor version of the Bak-Sneppen model can be well approximated by an exponential giving rise to a $1/f^2$ power spectrum.

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Recently, an exact solution of the random-neighbor version of the Bak-Sneppen model was presented by de Boer and co-workers [1]. They derived a master equation for the probability $P_n(t)$ that n of out of N numbers have a value less than a fixed value λ at (discrete) time t. In the limit $N\rightarrow\infty$ and $\lambda=1/2$, P_n has the scaling form $P_n(t)=(1/\sqrt{N})\ f(x=n/\sqrt{N},\ \tau=t/N)$. Inserting this expression into the master equation gives the following Fokker-Planck equation for $f(x,\tau)$ with a reflecting boundary at $x=0$:

$$\frac{\partial f}{\partial \tau}=\frac{1}{4}\frac{\partial^2 f}{\partial x^2}+\frac{\partial}{\partial x}(xf). \tag{1}$$

Consequently, the random-neighbor version of the Bak-Sneppen model for $N\rightarrow\infty$ is just an Ornstein-Uhlenbeck process, i.e., Brownian motion in a parabolic potential. Given the initial condition $f(x,0)=\delta(x-y)$, the solution is

$$f(x,\tau)=\sqrt{\frac{2}{\pi(1-\exp^{-2\tau})}}\exp\frac{-2(x-y\exp^{-\tau})^2}{1-\exp^{-2\tau}}. \tag{2}$$

It follows for the autocorrelation function $G(\tau)$ of the time signal $x(\tau)$

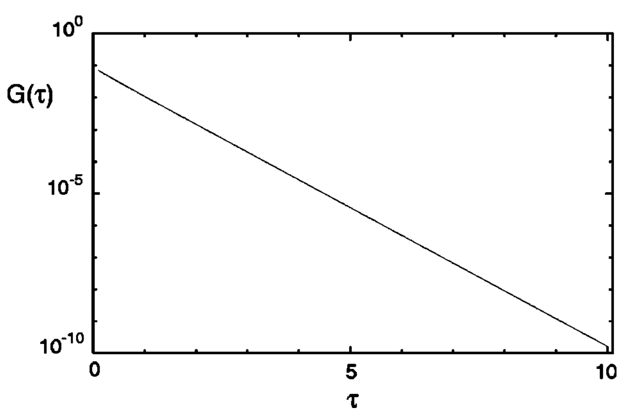

FIG. 1. Plot of the correlation function given by Eq. (6).

$$G(\tau)=\frac{1}{4}\exp-a|\tau|, \tag{3}$$

where $a\equiv 1$. This directly gives the power spectral density $S(\tilde{\omega})$ via a Fourier transform of $G(\tau)$

$$S(\tilde{\omega})=\frac{1}{2}\frac{a}{a^2+\tilde{\omega}^2}. \tag{4}$$

Going back to the unscaled variables leads to

$$S_P(\omega)=\frac{1}{2}\frac{a}{\dfrac{a^2}{N^2}+\omega^2}, \tag{5}$$

which is only valid for low frequencies. Hence, the power spectral density of the signal $n(t)$ decays as $1/f^2$ and for very low frequencies it even becomes constant. However, the above calculation was carried out without applying the boundary condition at $x=0$. Nevertheless, it is already clear from a physical point of view that the functional form of $G(\tau)$ will not change drastically by incorporating a reflecting boundary. This is supported mathematically by the fact that one simply has to use the method of images. This was done in Ref. [2] giving

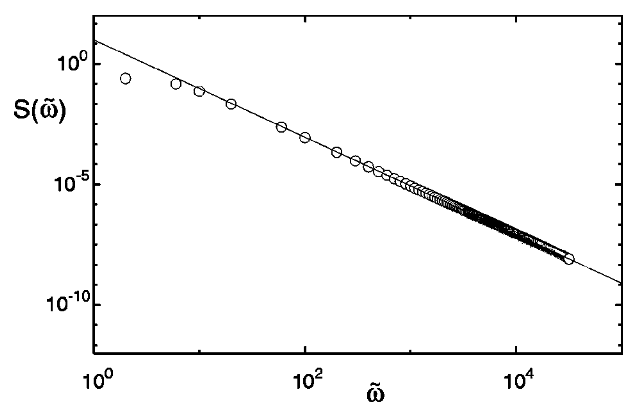

FIG. 2. Plot of the power spectrum given by a numerical Fourier transformation of Eq. (6). The solid line with exponent −2 is drawn for reference.

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$$G(\tau)=\frac{1}{8\pi}[1-\exp^{-2\tau}]^{3/2}\{F[1,2,3/2,r_-(\tau)]+F[1,2,3/2,r_+(\tau)]-F[1,2,5/2,r_-(\tau)]/3-F[1,2,5/2,r_+(\tau)]/3\}-\frac{1}{2\pi}, \tag{6}$$

where $F(a,b,c,z)$ is the hypergeometric function and where $r_\pm(\tau)=\frac{1}{2}[1\pm\exp(-\tau)]$. In Fig. 1, $G(\tau)$ is shown. We clearly find an exponential behavior with $a=0.869\pm0.008$ for $0.1<\tau<10$ giving rise to a power spectral density as in Eq. (4). This is confirmed by a numerical Fourier transform of Eq. (6) (see Fig. 2). Here, it has to be noted that $G(\tau)$ is an even function, i.e., $G(\tau)=G(-\tau)$. This also ensures that the Fourier transform $S(\tilde{\omega})$ is real.

In the case of the Bak-Sneppen model with one next neighbor, we also cannot confirm the results presented in Ref. [2]. A direct simulation of the time signal gives $S_P(\omega)\propto 1/\omega^{1.5}$ over 2 decades for a system size of $N=8192$. Hence, although the power spectral density in the Bak-Sneppen model decays as a power law, the exponent is far from one. This is also true for a different definition of the time signal [3].

In conclusion, there is no sign of $1/f$ noise in the random-neighbor version of the Bak-Sneppen model, and even in the next-neighbor version there isno $1/f$ noise in the strict sense.

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